Multi-Channel Beat-Frequency Digital Measurement System for Frequency Standard

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Abstract—**A multi-channel Beat-Frequency Digital Measurement system (BFDMS) using digital signal processing method has been developed and operated in NTSC. The system can high-precision analyze and monitor high-stability frequency sources. It consists of three independent measuring channels what can be operated simultaneously and one calibrating channel to correct the systematic error. This result that the measured noise floor Allan deviation is approximately 3e-14 at one second has been approved in system test.**

The key technique is the digital correlation algorithm that has been designed to reduce the effecting of beat-frequency device noise floor on measurement accuracy. So the limit factor of improving measurement precision that is brought by circuit noise of beat-frequency technology has been solved. The algorithm can calculate phase difference either. To improve ability of measurement, the system has a calibration channel what is designed to remove the systematic error. In this channel, the known reference frequency can be used to deduce systematic error. And then this result will be used to modify the value of measurement channels. The principles of operation, and error analysis have been verified will be introduced in detail in this article.

I. INTRODUCTION

High-precision frequency measurement techniques are important in any branch of science and technology such as radio astronomy, high-speed digital communications, and high-precision time synchronization. At present, the frequency stability of some of atomic oscillators is approximately 10E-16 at 1 second and there is no sufficient instrument to measure it.

Since direct frequency measurement methods is far away from the requirement of measurement high-precision oscillator, so the research of indirect frequency measurement methods are widely developed. Presently, common methods of measuring frequency include Dual-Mixer Time Difference (DMTD), Frequency Difference Multiplication (FDM), and Beat-Frequency (BF). DMTD is arguably one of the most precise ways of measuring an ensemble of clocks all having the same nominal frequency, because it can cancel out common error in the overall measurement process. FDM is one of the methods of high-precision measurement by multiplying frequency difference to intermediate frequency. Comparing with forenamed methods, the BF has an advantage that there is the simplest structure, and then it leads to the lowest device noise. However, the lowest device noise doesn't means the highest accuracy, because it sacrifices accuracy to acquire simple configuration. Therefore, the BF method wasn't paid enough attention to measure precise oscillators.

With studying the BF methods of measuring frequency, we conclude that the abilities of measuring frequency rest with accuracy of counter and noise floor of beat-frequency device. So designing a scheme that it can reduce circuit noise of beat-frequency device is mainly mission as the model of counter has been determined. As all well known, reducing circuit noise need higher technics to realize, and it is hardly and slowly, therefore, we need to look for another solution to improve the accuracy of BF method. In view of this reason, we design a set of algorithm to smooth circuit noise of beat-frequency device and realize the multi-channel Beat-Frequency Digital Measurement system (BFDMS) design goal of low noise floor.

II. PRINCIPLE OF BEAT-FREQUENCY

The BF method mixes (subtracts) the two sources being compared, and measures the period of the resulting low-frequency beat notes at frequency f_b as illustrated in figure 1. The signals from two independent sources are fed into the two ports of mixer. One of the sources runs at frequency $f_r = f_0 - f_b$ and it is generated by an Offset Generator (OG) with external reference frequency f_0. The other one running at frequency f_x is from under test source. The nominal frequency of under test source should be f_0, so the real frequency is referred as $f_x = f_0 + \Delta f$, where the frequency Δf is the value that needs to be measured actually. Therefore, the under test source will be down-converted to beat notes at frequency $f_b + \Delta f$ by mixer, and then measure the beat notes. Because the frequency of beat notes is less

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than under test source, we can use digital analysis tool, such as analog to digital converter, Digital Signal Processing device, to measure beat notes.

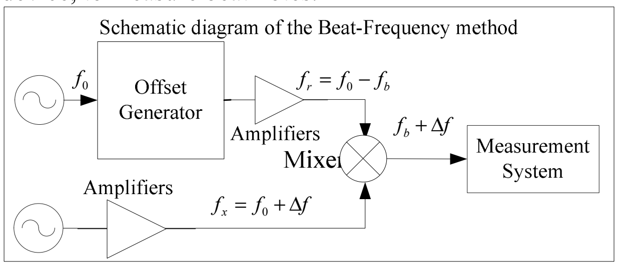

Fig.1. Schematic diagram of the Beat-Frequency

The BF method can increase measurement resolution by down-converting under test source with reference signal (Beat-Frequency factor the ratio of the reference to the beat frequency: f_0 / f_b). For example, mixing reference source at frequency $f_0 = 10MHz$ against $f_r = 10MHz - 1Hz$ offset reference from the OG will produce beat notes at frequency 1Hz whose period variations are enhanced by a factor of 10 MHz/1 Hz = 10^7. Thus, a time interval counter with 100 nanosecond resolution (10 MHz clock) can resolve clock phase changes of 10 femtosecond.

III. STRUCTURE OF BFDMS

The multi-channel Beat-Frequency Digital Measurement System consists of Multi-channel Beat-Frequency signal Generator (MBFG) and Digital Signal Processing (DSP) module. The multi-channel means three test channels and one calibration channel with same physical structure. The system block diagram is shown in figure 2.

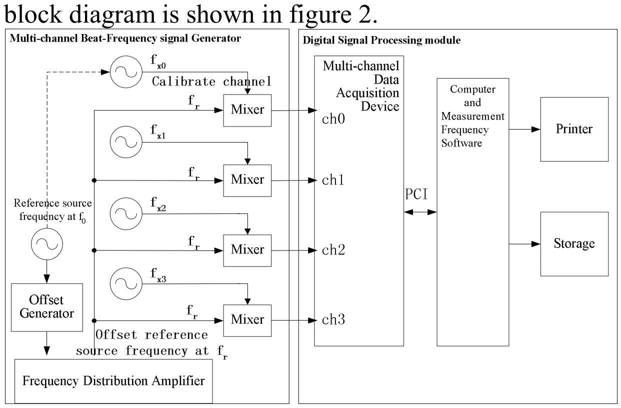

Fig.2. Block diagram of the multi-channel beat-frequency digital measurement system

The MBFG is made up of Offset Generator, Frequency Distribution Amplifier (FDA), and mixer. There are four signals input from outside sources. One of these is designed as the reference, generally chosen to be the most reliable source. The signal provides the reference f_0 for the OG and the input for channel 0. The reference source drives the OG, a synthesizer whose output frequency is set to $f_r = f_0 - f_b$. The OG output drives FDA to acquire four or more offset source at frequency f_r . Three sources under test, denoted frequency $f_{xi}, i = 1,2,3$, are down-converted to sinusoidal beat-frequency signal at frequency around f_b by mixing them with three offset reference sources.

Channel 0 requires an input signal that comes from the reference source running at frequency f_0 . This is used for the self-testing of the noise floor and calibrating systematic errors. In this one, the reference source is down-converted with offset source from the FDA, which is driven by the reference source. Because the two ports of mixer have been fed into corresponding signals from common source. Calibrating systematic errors can be realized by comparing the measurement result of this channel with the others.

The Digital Signal Processing module consists of multi-channel Data Acquisition device (DAQ), computer and output devices. The Measurement Frequency software is installed in PC to analyze data from DAQ. The beat notes, which are output from the MBFG that are connected to channels of analog-to-digital converter respectively, are digitized according to the same timing by the DAQ that are driven by a clock with sampling frequency N . Then, MF software retrieves the data from buffer of DAQ, maintains synchronization of the data stream, carries out processing of measurement (including frequency, phase difference, and analyzing stability), stores original data to disk, and manages the output devices.

The MBFG output must be sinusoidal beat-frequency signal, because processing beat-frequency signal makes use of the property of trigonometric function. It has the obvious difference with traditional beat-frequency method using square waveform and Zero Crosser Assembly.

IV. PRINCIPLES OF OPERATION

According to above introduction, the Δf could be acquired by measuring the beat-frequency signal at frequency $f_b + \Delta f$ because of the f_b known. We propose digital correlation technique to compute Δf . This BFDMS can be used to analyze not only frequency stability of atomic oscillators, but also phase difference. Following content will describe the algorithms using formula.

One of the MBFG outputs is expressed with the following beat-frequency signal $v_i(t)$:

$$v_i(t) = V_i \sin(2\pi(f_b + \Delta f_i)t + \varphi_i), i = 1,2,3 \quad (1)$$

Where V_i indicates amplitude of channel i, $f_b = f_0 - f_r$ is the nominal frequency of beat-frequency signal, unknown fractional frequency Δf_i of source under test in channel i consist of integral portion f_{int} and decimal portion f_{dec} and will be measured in this system, here $\Delta f_i = f_{\text{int}} + f_{dec}$, φ_i denotes the initial phase of channel i.

Since the test sources actually include random noise and quantization noise. So the beat-frequency signals that have been quantized can be expressed by following formula (2):

$$v_i'(n) = V_i \sin(2\pi \frac{f_b + \Delta f_i}{N} n + \varphi_i) + g_i(n) + l_i(n), n = 1,2,3,\cdots \quad (2)$$

Here N is sampling frequency of analog-to-digital converter (ADC), $g_i(n)$ denotes random noise of channel i, $l_i(n)$ is quantization noise of channel i and generates by ADC, n is a positive integer and its value is in the range $1 \sim \infty$. Formula

(2) could be transformed into following normalized expression (3) to deduce conveniently.

$$v_i(n) = \sin(2\pi \frac{f_b + \Delta f_i}{N} n + \varphi_i) + g_i(n) + l_i(n) \quad (3)$$

To realize one time frequency measurement, sampling beat-frequency signal must be continuous operated at least two seconds. For example, the j-th measurement frequency of channel i will analyze the j second $v_{ij}(n)$ and j+1 second $v_{i(j+1)}(n)$ data from DAQ.

Here, $\omega_{ij} = 2\pi \frac{f_b + \Delta f_{ij}}{N}$, $x_{ij}(n) = \sin(\omega_{ij} n + \varphi_{ij})$, $x_{i(j+1)}(n) = \sin(\omega_{ij} n + \varphi_{i(j+1)})$ is assumed. That is, $v_{ij}(n)$ according to formula (3) is given by

$$\begin{aligned} v_{ij}(n) &= \sin(\omega_{ij} n + \varphi_{ij}) + g_{ij}(n) + l_{ij}(n) \\ &= x_{ij}(n) + g_{ij}(n) + l_{ij}(n), i = 1,2,3. j = 1,2...\infty \end{aligned} \quad (4)$$

$$\begin{aligned} v_{i(j+1)}(n) &= \sin(\omega_{ij} n + \varphi_{i(j+1)}) + g_{i(j+1)}(n) + l_{i(j+1)}(n) \\ &= x_{i(j+1)}(n) + g_{i(j+1)}(n) + l_{i(j+1)}(n) \end{aligned} \quad (5)$$

Where the initial phase of the j second and j+1 second are denoted by φ_{ij} and $\varphi_{i(j+1)}$ respectively.

The cross-correlation between $v_{ij}(n)$ and $v_{i(j+1)}(n)$ have been used by following formula:

$$\begin{aligned} R_{ij}(m) &= \frac{1}{N}\sum_{n=0}^{N-1} v_{ij}(n) v_{i(j+1)}(n+m) \\ &= \frac{1}{N}\sum_{n=0}^{N-1} \begin{aligned}&[x_{ij}(n) + g_{ij}(n) + l_{ij}(n)] \\ &\times [x_{i(j+1)}(n+m) + g_{i(j+1)}(n+m) \\ &+ l_{i(j+1)}(n+m)]\end{aligned} \\ &= \frac{1}{2}\cos(\omega_{ij} m + \Phi_{ij}) + R_{x_{ij}g_{i(j+1)}} + R_{x_{ij}l_{i(j+1)}} + R_{g_{ij}x_{i(j+1)}} \\ &+ R_{g_{ij}g_{i(j+1)}} + R_{g_{ij}l_{i(j+1)}} + R_{l_{ij}x_{i(j+1)}} + R_{l_{ij}g_{i(j+1)}} + R_{l_{ij}l_{i(j+1)}} \end{aligned} \quad (6)$$

Here two series $v_{ij}(n)$ and $v_{i(j+1)}(n)$ where n=0,1,2…N-1 are carried out correlating operation. Where m denotes the delay and m=0,1,2…N-1, $\Phi_{ij} = \varphi_{i(j+1)} - \varphi_{ij}$ is the initial phase difference of corresponding series.

The cross-correlation function between $v_{ij}(n)$ and $v_{i(j+1)}(n)$ can be calculated as follow at delay m=0.

$$\begin{aligned} R_{ij}(0) &= \frac{1}{N}\sum_{n=0}^{N-1} v_{ij}(n) v_{i(j+1)}(n) \\ &= \frac{1}{2}\cos(\Phi_{ij}) + R_{x_{ij}g_{i(j+1)}} + R_{x_{ij}l_{i(j+1)}} + R_{g_{ij}x_{i(j+1)}} \\ &+ R_{g_{ij}g_{i(j+1)}} + R_{g_{ij}l_{i(j+1)}} + R_{l_{ij}x_{i(j+1)}} + R_{l_{ij}g_{i(j+1)}} + R_{l_{ij}l_{i(j+1)}} \end{aligned} \quad (7)$$

The information of Φ_{ij} will be discussed at section 6.

Formula (6) could be split into three parts; with the first part is cross-correlation function between signals $x(n)$:

$$A = \frac{1}{2}\cos(\omega_{ij} m + \Phi_{ij})$$

the second part is the cross-correlation function between noise and signal;

$$B = R_{x_{ij}g_{i(j+1)}} + R_{x_{ij}l_{i(j+1)}} + R_{g_{ij}x_{i(j+1)}} + R_{l_{ij}x_{i(j+1)}}$$

Where $g(n)$ and $l(n)$ are the random noise produced by MBFG and quantization noise produced by ADC respectively, which can be assumed that between noise and signal $x(n)$ is uncorrelation here. Then the value of the B can be zero. In actually, the B is nonzero. The influence will be taken into account in section 6.

the third part is the cross-correlation function between noise and noise:

$$C = R_{g_{ij}g_{i(j+1)}} + R_{g_{ij}l_{i(j+1)}} + R_{l_{ij}g_{i(j+1)}} + R_{l_{ij}l_{i(j+1)}}$$

According to the property of correlation function, if two circular signals are correlated then it will result in a circular signal with the same period as the original signal. Therefore, the C can be denoted average $R_{ij}(m)$ over m:

$$C = \frac{1}{N}\sum_{m=0}^{N-1} R_{ij}(m) \quad (8)$$

Then

$$\cos(\Phi_{ij}) = 2(R_{ij}(0) - \frac{1}{N}\sum_{m=0}^{N-1} R_{ij}(m)) \quad (9)$$

The initial phase difference Φ_{ij} is given as follows:

$$\Phi_{ij} = 2k\pi \pm \arccos(2(R_{ij}(0) - \frac{1}{N}\sum_{m=0}^{N-1} R_{ij}(m))), k = 1,2... \quad (10)$$

Consider the relationship between $v_{ij}(n)$ and $v_{i(j+1)}(n)$ where the initial phase of $v_{i(j+1)}(n)$ should equal to the phase of $v_{ij}(n)$ at n=N. This relationship can be given as follows:

$$\varphi_{i(j+1)} = \varphi_{ij} + 2\pi(\frac{f_b + \Delta f_{ij}}{N} \times N) \quad (11)$$

According to above-mentioned definition of f_b , it represents the nominal frequency of beat-frequency signal and is a known value. In view of $\Delta f_{ij} = f_{\text{int}} + f_{dec}$, and f_{int} is an integer. The decimal portion f_{dec} of fractional frequency Δf_{ij} is given by

$$\begin{aligned} &\varphi_{i(j+1)} = \varphi_{ij} + 2\pi\Delta f_{ij} \\ &\Rightarrow \Phi_{ij} = \varphi_{i(j+1)} - \varphi_{ij} = 2\pi(f_{\text{int}} + f_{dec}) \\ &\Rightarrow \Phi_{ij} = 2\pi f_{dec} \end{aligned} \quad (12)$$

If the Eqs. $f_{\text{int}} = 0$ hold ,then the Eqs. $f_{dec} = \Delta f_{ij}$ hold. We have obtained the fractional frequency Δf_{ij} . Additional knowledge about the nominal frequency f_0 of under test signal would let us obtain the actual frequency of under test signal $f_{x_{ij}} = f_0 + \Delta f_{ij}$ in channel i.

If the f_{int} is unzero, then the Δf_{ij} can be obtained by following formula.

The cross-correlation R_{ij} at delay m=1 is defined as

$$
\begin{aligned}
R_{ij}(1) &= \frac{1}{N}\sum_{n=0}^{N-1} v_{ij}(n) v_{i(j+1)}(n+1) \\
&= \frac{1}{2}\cos(\omega_{ij} + \Phi_{ij}) + R_{x_{ij}g_{i(j+1)}} + R_{x_{ij}l_{i(j+1)}} \\
&+ R_{g_{ij}x_{i(j+1)}} + R_{g_{ij}g_{i(j+1)}} + R_{g_{ij}l_{i(j+1)}} \\
&+ R_{l_{ij}x_{i(j+1)}} + R_{l_{ij}g_{i(j+1)}} + R_{l_{ij}l_{i(j+1)}} \\
&= \frac{1}{2}\cos(2\pi\frac{f_b + \Delta f_{ij}}{N} + \Phi_{ij}) + C
\end{aligned}
\quad (13)
$$

The Δf_{ij} is computed according to the following formula from the C in formula (8) and the Φ_{ij} in formula (10):

$$
\cos(2\pi\frac{f_b + \Delta f_{ij}}{N} + \Phi_{ij}) = 2R_{ij}(1) - C \qquad (14)
$$

Here, f_b is known, and the Eqs. $N >> f_b$ hold. N is the sampling frequency of ADC.

The systematic errors from MBFG are extracted by measuring the calibration channel, and then it is used to modify the results of frequency measurement. Based on the same processing, this algorithm can be used to measure the phase difference and so on.

V. SOFTWARE DESCRIPTION

The Measurement Frequency software (MF) of the BFDMS is operated by the LabWindows/CVI applications. MF configures the parameters of DAQ, stores original data and results of measuring to disk, maintains synchronization of the data stream, carries out the algorithms of measuring frequency and phase difference, analyzes frequency stability, retrieves the stored data from disk and prepares plots of original data, frequency, phase difference, and Allan deviation. Figure 3 shows the main interface window. To view interesting data, the user can implement the switch of tab panel control button between different functions.

MF consists of four applications, a virtual instrument panel that is the user interface to control the hardware and the others via DLL, a server program that manages the data, a processing program, and a output program that prepares plots of results of measuring or exports by computer interface. Figure (4) shows the block diagram.

The virtual instrument panel can build path between the icons and program or between the instrument and user. It consists of options pull-down menu, functions button, tab panel, where the tab panel are used to show information that include original data, frequency value, phase difference. Figure 3 (a) shows the window of choosing channels and (b) shows the strip chart of real-time original data of one of channels. Figure 3 (c,d) shows the real-time results of measuring by running the controls of tab panel.

Server program configures the parameters of each channel, maintains synchronization of the data stream, carries out the simple preprocessing (either ignore those points that are significantly less than or greater than the threshold or detect missing points and substitute extrapolated values to maintain data integrity), stores original data and results of measuring to disk.

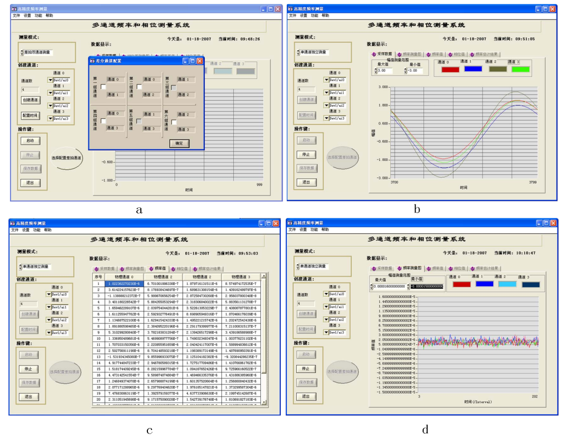

Fig. 3. MF of the BFDMS, (a) shows the window of choosing channels, (b) shows the strip chart of real-time original data of one of channels, (c) shows the lists of the real-time results of measuring, (d) shows strip chart of fractional frequency

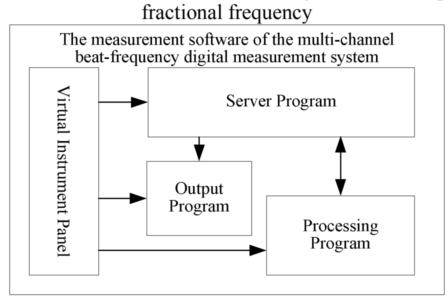

Fig.4 Block Diagram of the Measurement Software

Processing program retrieves the stored data from disk and carries out the processing. To run the processing program, the user chooses channel mode on the panel, choosing single channel mode gives single frequency results of three channels; choosing difference channel mode gives phase difference between two channels. As the single channel mode is chose, the Allan deviation can be given by carrying out processing program too. All the results that have been calculated will be stored to disk.

The output program manage the interface that communicate with other instruments, exports data that the user interests from disk and prepares plots of original data, frequency, phase difference and Allan deviation. Text files of these data are available if the user wishes to make further analyses.

VI. SYSTEM NOISE FLOOR

The main principle of operation of BFDMS is digital correlation algorithms, which can reduce the effects of circuit noise floor and improve the measurement precision. In addition, this system is more reliability and maintainability because the structure of system is simpler than other high-precision frequency measurement system. This section will discuss the noise floor of the proposed system.

The sources of errors include quantization noise and random disturbance noise in digital frequency measurement processing.

Here, quantization noise is generally caused by the nonlinear transmission of AD converter. To analysis the noise, AD conversion usual is regarded as a nonlinear mapping from the continuous amplitude to quantization amplitude. The error that is caused by the nonlinear mapping can be calculated by using either the random statistical approach or nonlinear determinate approach. The random statistical approach means that the results of AD conversion are expressed with the sum of sampling amplitude and random noise, and it is the major approach to calculate the error at present.

Although random disturbance noise can be removed by running digital correlation algorithms, finite number of sampling data have to be used to instead of White Gaussian noise in practical instances, and quantization noise that is uniformly distributed, which will lead to the results that the cross-correlation between the signal and noise aren't completely uncorrelated. Then the term $B=R_{x_{ij}g_{i(j+1)}}+R_{x_{ij}l_{i(j+1)}}+R_{g_{ij}x_{i(j+1)}}+R_{l_{ij}x_{i(j+1)}}$ of cross-correlation can't be ignored. Because the term B isn't strictly zero. Considering it has been ignored in section 4 to analysis conveniently. We will discuss the effect of ignoring B and C on measurement precision in following paper.

$$B=R_{x_{ij}g_{i(j+1)}}+R_{x_{ij}l_{i(j+1)}}+R_{g_{ij}x_{i(j+1)}}+R_{l_{ij}x_{i(j+1)}} \tag{15}$$

According to the property of cross-correlation and sine function, we have

$$\begin{aligned}R_{x_{ij}g_{i(j+1)}}(m)&=R_{g_{i(j+1)}x_{ij}}(-m)=\frac{1}{N}\sum_{n=0}^{N-1}g_{i(j+1)}(n)x_{ij}(n-m)\\&=\frac{1}{N}\sum_{n=0}^{N-1}g_{i(j+1)}(n)\sin(\varphi_{ij}+\omega_{ij}n-\omega_{ij}m)\\&=\frac{1}{N}\sum_{n=0}^{N-1}g_{i(j+1)}(n)[\sin(\varphi_{ij}+\omega_{ij}n)\cos(\omega_{ij}m)-\cos(\varphi_{ij}+\omega_{ij}n)\sin(\omega_{ij}m)]\\&=\frac{1}{N}\cos(\omega_{ij}m)\sum_{n=0}^{N-1}g_{i(j+1)}(n)\sin(\varphi_{ij}+\omega_{ij}n)\\&\quad-\frac{1}{N}\sin(\omega_{ij}m)\sum_{n=0}^{N-1}g_{i(j+1)}(n)\cos(\varphi_{ij}+\omega_{ij}n)\end{aligned} \tag{16}$$

Similarly, for other cross-correlation, we have

$$\begin{aligned}R_{x_{ij}l_{i(j+1)}}(m)&=\frac{1}{N}\sum_{n=0}^{N-1}l_{i(j+1)}(n)x_{ij}(n-m)\\&=\frac{1}{N}\cos(\omega_{ij}m)\sum_{n=0}^{N-1}l_{i(j+1)}(n)\sin(\varphi_{ij}+\omega_{ij}n)\\&\quad-\frac{1}{N}\sin(\omega_{ij}m)\sum_{n=0}^{N-1}l_{i(j+1)}(n)\cos(\varphi_{ij}+\omega_{ij}n)\end{aligned} \tag{17}$$

$$\begin{aligned}R_{g_{ij}x_{i(j+1)}}(m)&=\frac{1}{N}\sum_{n=0}^{N-1}g_{ij}(n)x_{i(j+1)}(n+m)\\&=\frac{1}{N}\cos(\omega_{ij}m)\sum_{n=0}^{N-1}g_{ij}(n)\sin(\varphi_{i(j+1)}+\omega_{ij}n)\\&\quad+\frac{1}{N}\sin(\omega_{ij}m)\sum_{n=0}^{N-1}g_{ij}(n)\cos(\varphi_{i(j+1)}+\omega_{ij}n)\end{aligned} \tag{18}$$

$$\begin{aligned}R_{l_{ij}x_{i(j+1)}}(m)&=\frac{1}{N}\sum_{n=0}^{N-1}l_{ij}(n)x_{i(j+1)}(n+m)\\&=\frac{1}{N}\cos(\omega_{ij}m)\sum_{n=0}^{N-1}l_{ij}(n)\sin(\varphi_{i(j+1)}+\omega_{ij}n)\\&\quad+\frac{1}{N}\sin(\omega_{ij}m)\sum_{n=0}^{N-1}l_{ij}(n)\cos(\varphi_{i(j+1)}+\omega_{ij}n)\end{aligned} \tag{19}$$

Then, the B can be obtained as follows:

$$\begin{aligned}B&=\frac{1}{N}\cos(\omega_{ij}m)[\sum_{n=0}^{N-1}g_{i(j+1)}(n)\sin(\varphi_{ij}+\omega_{ij}n)+\sum_{n=0}^{N-1}l_{i(j+1)}(n)\sin(\varphi_{ij}+\omega_{ij}n)\\&+\sum_{n=0}^{f_s-1}g_{ij}(n)\sin(\varphi_{i(j+1)}+\omega_{ij}n)+\sum_{n=0}^{f_s-1}l_{ij}(n)\sin(\varphi_{i(j+1)}+\omega_{ij}n)]\\&+\frac{1}{N}\sin(\omega_{ij}m)[\sum_{n=0}^{N-1}g_{ij}(n)\cos(\varphi_{i(j+1)}+\omega_{ij}n)-\sum_{n=0}^{N-1}g_{i(j+1)}(n)\cos(\varphi_{ij}+\omega_{ij}n)\\&+\sum_{n=0}^{N-1}l_{ij}(n)\cos(\varphi_{i(j+1)}+\omega_{ij}n)-\sum_{n=0}^{N-1}l_{i(j+1)}(n)\cos(\varphi_{ij}+\omega_{ij}n)]\end{aligned} \tag{20}$$

The sum of formula (20) is equal to zero in the range [0,N-1].

$$\sum_{m=0}^{N-1}B=0 \tag{21}$$

In view of the Eq. (21), although the B isn't strictly zero, their sum is equal to zero. We all known that on the right-hand side of Eq.(8) is the sum of cross-correlation function. Applying the Eq. (21) to (8) term by term, we obtain that the Eq.(8) strictly hold. Now we have the knowledge that the term C doesn't effect on the measurement results and we just need to discuss the term B as follows. Eq. (9) can be given by

$$R_{ij}(0)-\frac{1}{N}\sum_{m=0}^{N-1}R_{ij}(m)=\frac{1}{2}\cos(\Phi_{ij})+B \tag{22}$$

Let the error terms that are caused by the white Gaussian noise and the quantization noise be represented by $B_1=R_{x_{ij}g_{i(j+1)}}+R_{g_{ij}x_{i(j+1)}}$ and $B_2=R_{x_{ij}l_{i(j+1)}}+R_{l_{ij}x_{i(j+1)}}$ respectively. Then $B=B_1+B_2$.

We assume that $g(t)$ is Gaussian random variable of mean '0'and standard deviation 'σ_g^2'. In the view of Eq.(16) and (18), we have obtained the standard deviation as follow:

$$\sigma_{B_1}^2=\frac{2\sigma_g^2}{N} \tag{23}$$

Assume that the AD converter is round-off uniformly quantizer and using quantization step Δ. Then $l(t)$ is uniformly distributed in the range $\pm\Delta/2$ and its mean value is zero and standard deviation is $(\Delta^2/12)$. According to the Eq.(14) and (19), we have $\sigma_{B_2}^2=\frac{2\Delta^2}{12N}$ (24)

For B_1 and B_2 are uncorrelated, then

$$\sigma_B^2=\sigma_{B_1}^2+\sigma_{B_2}^2=\frac{2\sigma_g^2}{N}+\frac{2\Delta^2}{12N} \tag{25}$$

The mean square value of $\frac{1}{2}\cos(\Phi_{ij})+B$ on the right-hand side of formula (22) will be calculated by the following formula to evaluate the influence of noise on measurement initial phase difference.

$$\frac{1}{N}\sum_{m=0}^{N-1}(\frac{1}{4}\cos^2(\Phi_{ij})+B\cos(\Phi_{ij})+B^2)$$
$$=\frac{1}{4}\cos^2(\Phi_{ij})+\frac{1}{N}\sum_{m=0}^{N-1}(B\cos(\Phi_{ij})+B^2)$$
$$=\frac{1}{4}\cos^2(\Phi_{ij})+(\frac{2\sigma_g^2}{N}+\frac{2\Delta^2}{12N})+\frac{1}{N}\sum_{m=0}^{N-1}B\cos(\Phi_{ij}) \qquad (26)$$
$$\le\frac{1}{4}\cos^2(\Phi_{ij})+(\frac{2\sigma_g^2}{N}+\frac{2\Delta^2}{12N})+\frac{1}{N}\sum_{m=0}^{N-1}B$$
$$=\frac{1}{4}\cos^2(\Phi_{ij})+(\frac{2\sigma_g^2}{N}+\frac{2\Delta^2}{12N})$$

Where σ_g^2 represent standard deviation of Gaussian random variable, Signal Noise Ratio $SN=\frac{V^2}{\sigma_g^2}$, and here the V is the amplitude of input signal, let amplitude resolution of a-bit digitize and quantization step be Δ, here variable 'a' can be 8~24. We have $\frac{\Delta}{V}=\frac{2}{2^a-1}$. Applying this equation to formula (26) term by term, we obtain

$$\sigma_e=\sqrt{\frac{1}{4}\cos^2(\Phi_{ij})+\frac{1}{N}(\frac{2V^2}{SN^2}+\frac{2\Delta^2}{12})} \qquad (27)$$

Where the σ_e is the standard deviation of measurement initial phase difference. The standard deviation of digital correlation algorithms depends on the sampling frequency N, SNR and amplitude resolution 'a', as understood from formula (27). Here the noise of amplitude resolution can be ignored if the 'a' is sufficiently bigger than 16-bit and the SNR is smaller than 100 dB. The measurement accuracy for this method is mostly related to SNR of signal. This method has been tested that has the strong anti-disturbance capability.

VII. Noise Floor Observations and Conclusions

To evaluate the noise floor, we designed the platform when the test signal and reference signal were distributed in phase from a single signal generator. The signal generator at 10MHz and the beat-frequency value of 1Hz were set. For this example the Allan deviation at 1 second is 3×10^{-14}.

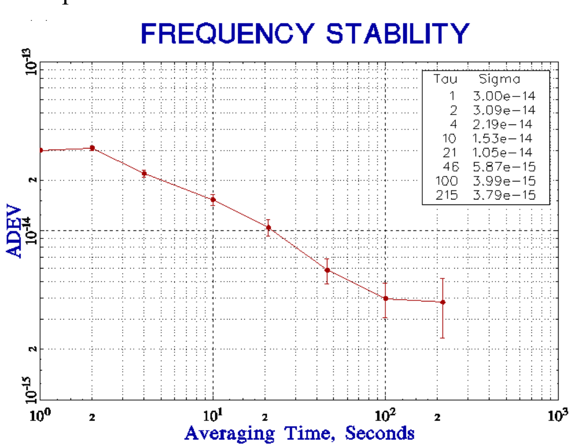

Fig.5. An example of noise floor characteristics of the BFDMS: Allan deviation

The measurement ability could be optimized further by improving the performance of OG. Because the reference of the system is drove by the output of OG.

Since the digital correlation techniques can smooth the effects of random disturbance of the MBFG, it can achieve higher measurement accuracy than other methods even if on the same MBFG.

Additional, the design of calibration channel that is proposed to remove the systematic error is useful to acquire better performance for current application. A comprehensive set of noise floor tests under all conditions has not been carried out with the current system.

The system hardware consists only of MBFG, DAQ and PC. Compared with the conventional systems using counter and beat-frequency device, the system can be miniaturized and moved conveniently. As expected, system noise floor is good enough for current test requirement. The system will take measurement of wide range frequency into account in the future. Intuitive operator interface and command remotely will be design in following work.

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